

RECEIVED: June 21, 2007 REVISED: January 22, 2008 ACCEPTED: March 20, 2008 PUBLISHED: April 14, 2008

# Warped supersymmetry breaking

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ABSTRACT: We address the size of supersymmetry-breaking effects within string theory settings where the observable sector resides deep within a strongly warped region, with supersymmetry breaking not necessarily localized in that region. Our particular interest is in how the supersymmetry-breaking scale seen by the observable sector depends on this warping. We focus concretely on type IIB flux compactifications and obtain this dependence in two ways: by computing within the microscopic string theory supersymmetry-breaking masses in Dp-brane supermultiplets; and by investigating how warping gets encoded into masses within the low-energy 4D effective theory. We identify two different ways to identify 'the' 4D gravitino in such systems — the state whose supersymmetry is the least broken, and the state whose couplings are the most similar to the 4D graviton's — and argue that these need not select the same state in strongly warped settings. We formulate the conditions required for the existence of a description in terms of a 4D SUGRA formulation, or in terms of 4D SUGRA together with soft-breaking terms, and describe in particular situations where neither exist for some non-supersymmetric compactifications. We suggest that some effects of warping are captured by a linear A dependence in the Kähler potential. We outline some implications of our results for the KKLT scenario of moduli stabilization with broken SUSY.

Keywords: Supersymmetry Breaking, Flux compactifications.

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# 1. Introduction

Understanding how supersymmetry breaks has been a Holy Grail for string theorists for decades, because it is likely a crucial prerequisite for understanding string theory's low-energy predictions. Like the search for the Grail this has proven to be an elusive quest, complicated as it is by related issues of modulus stabilization. Considerable progress has come recently, however, with the recognition that Type IIB string vacua can stabilize many moduli in the presence of fluxes [1, 2].

The remarkable properties of the warped compactifications to which these studies lead open up potentially interesting new possibilities for constructing phenomenologically attractive string vacua, both for applications to particle physics [3] and to cosmology [4].

They do so for several reasons. First, by providing a plausible setting in which all moduli may be fixed [2, 5] they provide a concrete laboratory within which to compute how supersymmetry breaks. This potentially represents a great leap forward since it allows one to deal with a serious drawback of previous calculations of supersymmetry-breaking effects for string vacua (for a review see [6]). Because these earlier calculations did not construct the potential which stabilized the various moduli, they could not determine the values of the moduli. In particular, they could not address whether or not supersymmetry was restored at a minimum of the moduli potential, as is very often the case in practice.

The second reason these warped compactifications have been so intriguing for phenomenology is that they can contain strongly-warped regions ('throats') within which the energies of localized states can be strongly suppressed by gravitational redshift. This possibility is very interesting since the energy gained by localization can dominate the energy cost due to the gradients which the localization requires. This can dramatically change the kinds of states which dominate the low-energy world, with localized states often being preferred over those which spread to fill all of the compact dimensions. Among the consequences of this observation is the possibility of having new ways to obtain large hierarchies of scale, such as by having all Standard Model degrees of freedom localized in a region of strong warping [7-10]. It also opens up new ways for energy to be efficiently channelled into such throats, potentially providing new ways to think about reheating during early-universe cosmology [11, 12].

Our goal in this paper is to analyze the size of supersymmetry-breaking effects within highly-warped compactifications of string theory, with emphasis on when the low-energy physics can be captured by an effective 4D description, and on how this description depends on the underlying scales. We focus on supersymmetry breaking triggered by nonzero (0,3) bulk fluxes within Type IIB vacua (for related work in this direction see [13]), and where necessary we imagine matter fields are identified with open-string degrees of freedom in the world-volume of D-branes, being suitably described at low energies by the corresponding Dirac-Born-Infeld and Chern-Simons actions. Within this framework we follow how supersymmetry breaking depends on three important parameters:  $\vartheta$  the strength of supersymmetry breaking fluxes;  $^1e^{A_m}$  the minimal value of warping, as well as the compactification volume,  $\mathcal{V} = V/\alpha'^3$ .

While reproducing standard results in the limit of small warping, we find several interesting differences when the warping is large. In particular, two natural criteria for identifying 'the' gravitino of the low-energy 4D theory point to different states. The first of these, the criterion of the least broken supersymmetry, selects the lightest of all gravitino Kaluza-Klein (KK) modes (or equivalently, the gravitino state which becomes massless if supersymmetry breaking is adiabatically switched off). In the presence of strong warping we argue that the wave-function for this state is localized within the strongly warped region, as is generically true for KK modes [14–16]. However, because of this strong warping the strength of the interactions of this state are generically set by a warped mass scale, which can be much smaller than the 4D Planck mass,  $M_p$ . This is what makes this state

 $<sup>{}^{1}\</sup>vartheta$  will correspond to  $W_{0}$  in the regime that a 4D effective description is valid.

differ in general with the state selected by the second criterion: that whose interactions are most similar to the massless 4D graviton.

What we find echoes what is known about supersymmetry breaking in Randall-Sundrum geometries, as has been extensively studied in the context of  $S^1/\mathbb{Z}_2$  orbifold compactifications of 5D gauged supergravity on AdS backgrounds [17–19]. In these setups, the observable (Standard Model) sector generically resides on the infrared brane, at the strongly warped end of the orbifold, and supersymmetry breaking is mediated by the radial modulus and/or the Weyl anomaly. Both of the gravitino states described above have a simple physical interpretation in these models when the dual CFT description of the throat is used, with the lightest gravitino describing a spin-3/2 resonance of the strongly interacting CFT dynamics (which gauges an emergent supersymmetry, not simply related to the supersymmetry of the constituent states). In this instance it is the second criterion which gives the gravitino which partners the graviton in its couplings to the 'constituent' CFT degrees of freedom.

Although we argue that generically a 4D description need not be possible for strongly warped systems, when it is we find that strong warping alters the resulting low-energy Kähler potential for Type IIB string compactifications. We identify a universal contribution of this kind, which leads to exponential suppressions for F-terms in the low-energy theory. This is similar to what is seen for supersymmetric RS systems [20], where more explicit forms for the warping-induced modifications to the Kähler potential are possible because of the simplicity of the underlying 5D AdS geometry.

Our presentation comes in two parts. In the next section, 2, we provide a generic semi-quantitative discussion of the scales which arise in supersymmetry-breaking compactifications with warped extra dimensions. (Although some estimates and explicit calculations of supersymmetry breaking scales have been made elsewhere [21], these focus on only one of the two types of 4D gravitino mentioned above.) This section also gives insights on how the lightest gravitino mass undergoes warping suppression, illustrating the phenomenon with the aid of a scalar field toy model. We provide here a criterion for when to expect the low-energy limit to be described by a 4D effective theory, and when this effective theory should be described by a 4D supergravity, by a supergravity supplemented by soft-breaking terms, or by a generic non-supersymmetric field theory. Section 3 then follows with simple calculable examples of flux-induced mass terms for D3 and D7 branes, together with a discussion of an approach to summarizing the warping dependence of the mass in the low-energy 4D effective field theory. We describe aspects of phenomenology in § 4, and then close in § 5 with a short summary and some concluding remarks.

# 2. Mass scales and effective descriptions

We now turn to a qualitative description of the scales which arise when higher-dimensional supergravities are compactified on strongly warped geometries.<sup>2</sup> With a view to phe-

<sup>&</sup>lt;sup>2</sup>For some earlier general discussion of scales see [21]. For some discussion of the problems that arise in trying to derive an N=1 SUGRA potential in the context of non-trivial warping see [19, 22] and [23]; for discussion of a prescription to derive the potential and other aspects of the effective theory see [15].

nomenological applications, our interest is in particular on the scales and the effective field theory which are relevant to the 4D dynamics of the low-energy theory (possibly associated with particles which are localized deep within a strongly warped region), with supersymmetry broken in some other sector due to fluxes or some other supersymmetry-breaking effects.

We start with a general description of the scales and effective theories which can arise within strongly warped compactifications, before returning later to describe their relevance for the scale of supersymmetry breaking as seen by observers localized in the warped regions.

#### 2.1 Mass scales in warped throats

Since many of the issues which arise for supersymmetry breaking in warped environments are generic to the kinematics of warping, we start here by summarizing some general properties of warped compactifications, together with examples from IIB compactifications.

#### 2.1.1 KK excitations and scales

It is useful to begin with a reminder of the mass scales which arise within unwarped string compactifications, having a product-space vacuum configuration

$$ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu} + g_{mn}(y) dy^m dy^n, \qquad (2.1)$$

together with configurations for the other bosonic supergravity fields. Our conventions are to use 10D indices  $M, N, \ldots = 0, \ldots, 9$  for spacetime; 4D indices  $\mu, \nu, \ldots = 0, \ldots, 3$  for the observed (noncompact) dimensions; and 6D indices  $m, n, \ldots = 4, \ldots, 9$  for the hidden (compact) dimensions.

For simplicity we restrict ourselves to geometries,  $g_{mn}$ , whose volume and curvatures are all characterized by a single scale:  $M_{KK} = 1/L$ . Control over semiclassical calculations typically requires both a small dilaton,  $g_s = e^{\phi} \ll 1$ , and small curvatures,  $\alpha'/L^2 \ll 1$ , and so  $M_{KK} \ll M_s$  where  $M_s^2 = 1/\alpha'$  denotes the string scale. At scales below  $M_s$  massive string modes can be integrated out, leaving an effective-field-theory description in terms of a higher-dimensional supergravity.

When higher-dimensional fields are dimensionally reduced on such a space their linearized fluctuations about this background are expanded in a complete set of Kaluza Klein (KK) eigenmodes, e.g.

$$\delta\phi(x,y) = \sum_{k} \varphi_k(x) u_k(y), \qquad (2.2)$$

with mode functions  $u_k(y)$  chosen as eigenfunctions of various differential operators arising from the extra-dimensional kinetic operators,  $\Delta u_k = \lambda_k u_k$ . The differential operator is chosen so that the resulting 4D fields,  $\varphi_k(x)$ , satisfy the appropriate field equations for a particle having a mass  $m_k$ , which is computable in terms of the corresponding eigenvalue  $\lambda_k$ . This produces a tower of states having masses ranging between 0 and  $\infty$  (for marginally stable vacua), which are split in mass by  $\Delta m_k \sim O(M_{KK})$ .

The effective field theory describing energies smaller than  $M_{KK}$  is a four dimensional one, because there is insufficient energy to excite KK modes which can probe the extra

dimensions. Conversely, although it is possible to think of the effective theory above  $M_{KK}$  as a complicated 4D theory involving KK modes and strong couplings, it is better to think of it as being higher dimensional for several reasons, not least of which being that it is much simpler to do so. Moreover, it is also true that the KK modes have masses reaching right up to the UV cutoff, and so there is never a parametrically wide gap between the UV cutoff and the mass of the heaviest mode. Furthermore, the 4D picture obscures higher-dimensional spacetime symmetries, like general covariance and supersymmetry, whose presence is crucial to the consistency of the theory.

For warped configurations the main difference is that the vacuum metric takes the more general form

$$ds^{2} = e^{2A(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + g_{mn}(y) dy^{m} dy^{n}, \qquad (2.3)$$

where the warp factor  $e^{A(y)}$  varies in the extra dimensions. The physical interpretation of the warp factor can be found by considering the energy of a test particle having mass m and proper velocity  $u^M$ . If the particle is stationary at a specific point,  $y_0$ , in the extra dimensions then the normalization condition,  $u^2 = -1$ , implies  $u^M = \delta_0^M e^{-A(y_0)}$ . The four-dimensional energy of such a particle is then  $E = -m \xi_M u^M = e^{2A(y_0)} u^0 m = e^{A(y_0)} m$ , where  $\xi^M = \delta_0^M$  is the timelike Killing vector field corresponding to time translation. We see that the energy of such a test particle is highly suppressed in strongly warped regions where  $e^A \ll 1$ .

The KK reduction of fluctuations about such a space takes a form similar to eq. (2.2), but with mode functions which diagonalize different differential operators and satisfy different normalization conditions than in the unwarped case. For instance, a higher-dimensional scalar fluctuation of ten dimensional mass  $m_{10}$ , satisfies the equation

$$(\Box_{10} - m_{10}^2)\Phi = \left[e^{-2A}\eta^{\mu\nu}\partial_{\mu}\partial_{\nu} + \frac{e^{-4A}}{\sqrt{g}}\partial_{m}\left(\sqrt{g}e^{4A}g^{mn}\partial_{n}\right) - m_{10}^2\right]\Phi = 0.$$
 (2.4)

Four-dimensional KK masses are given by eigenvalues of the operator

$$\Delta = -\frac{e^{-2A}}{\sqrt{g}}\partial_m(\sqrt{g}e^{4A}g^{mn}\partial_n) + e^{2A}m_{10}^2, \tag{2.5}$$

rather than operator  $\Delta_0 = -(1/\sqrt{g})\partial_m(\sqrt{g}g^{mn}\partial_n) + m_{10}^2$  which would have been used in the absence of warping. Notice one effect of warping is to convert the 10D mass  $m_{10}$ , into a potential  $e^{2A(y)}m_{10}^2$  for the wavefunctions of the KK excitations. For a wavefunction localized in a region of large warping, with the local warp factor  $e^A \ll 1$ , one can expect a redshifted four dimensional mass  $m \sim m_{10}e^A$  in accord with the preceding discussion.

In type IIB string theory, it is possible to write explicit solutions for warped compactifications [2]. We shall use these solutions as our laboratory to study the effects of warped throats on supersymmetry breaking. First, we review some features of these compactifications which shall be relevant for our discussion.

In these constructions, the geometry takes the form

$$ds_{10}^2 = e^{2A} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + e^{-2A} \tilde{g}_{mn}(y) dy^m dy^n,$$
(2.6)

where  $\tilde{g}_{mn}$  is a metric on Calabi-Yau of fiducial volume  $\tilde{V} = {\alpha'}^3$ . With this metric the generic 4D KK masses for a scalar field satisfying  $(-\Box_{10} + m_{10}^2)\Phi = 0$  are given by the eigenvalues of

$$\Delta = -\frac{e^{4A}}{\sqrt{\tilde{g}}} \partial_m (\sqrt{\tilde{g}} \tilde{g}^{mn} \partial_n) + e^{2A} m_{10}^2, \tag{2.7}$$

rather than eq. (2.5).

The warp factor satisfies the equation of motion

$$-\tilde{\nabla}^2 e^{-4A} = \frac{G_{mnp}\bar{G}^{mnp}}{12\,\text{Im}\,\tau} + 2\kappa_{10}^2 T_3 \tilde{\rho}_3^{loc}$$
 (2.8)

where  $\tau = C_0 + ie^{-\phi}$  is the axio-dilaton,  $G_3 = F_3 - \tau H_3$  is the complex three-form field strength, the tilde indicates indices raised with  $\tilde{g}^{mn}$ , and  $\tilde{\rho}_3^{loc}$  represents localized sources of D3-brane charge.

Given a particular solution  $e^{-4A_0}$  of (2.8), we can always find a family of solutions [15] with parameter c,

$$e^{-4A} = e^{-4A_0} + c. (2.9)$$

One convenient choice is to take the particular solution to be orthogonal to the zero mode c,

$$\int d^6 y \sqrt{\tilde{g}} \, e^{-4A_0} = 0 \,, \tag{2.10}$$

which emphasizes that  $e^{-4A_0}$  cannot then be everywhere positive. This agrees with the statement that this quantity may become negative in regions of string-size around negative tension objects, where the supergravity approximation fails.

For large c, in most parts of the manifold, the warp factor is approximately constant and the background resembles that of a standard Calabi-Yau compactification. This Calabi-Yau like region is often referred to as the bulk. In this case c sets the scale of the metric over much of the bulk and so controls the overall size of the compactification, with

$$V = \int d^6 y \sqrt{g_6} \approx c^{3/2} \int d^6 y \sqrt{\tilde{g}_6} = c^{3/2} \tilde{V}, \qquad (2.11)$$

and so  $c \approx (V/\tilde{V})^{2/3} \propto \mathcal{V}^{2/3}$ , where V is the volume of the internal metric and  $\mathcal{V}$  the dimensionless volume relative to the string scale.

For later use, we also note the form of the metric associated with the four-dimensional Einstein frame (which is related to (2.6) by rescaling the noncompact dimensions):

$$ds_{10}^2 = \lambda [e^{-4A_0} + c]^{-1/2} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + [e^{-4A_0} + c]^{1/2} \tilde{g}_{mn}(y) dy^m dy^n, \qquad (2.12)$$

with

$$\frac{1}{\lambda} = \frac{1}{\tilde{V}} \int d^6 y \sqrt{\tilde{g}} (c + e^{-4A_0}), \qquad (2.13)$$

chosen to ensure a canonical 4D Einstein-Hilbert action. With the choice (2.10) this simplifies to

$$\lambda = \frac{1}{c} \propto \mathcal{V}^{-2/3} \ . \tag{2.14}$$

For generality we often quote results for arbitrary  $\lambda$ , bearing in mind the specialization to Einstein frame through (2.13).

As shown in [2], regions of strong warping can arise in Type IIB vacua from typical values of the flux quantum numbers. For instance, if M units of R-R flux and K units of NS-NS flux wrap the A and B cycles of a conifold locus respectively, the solutions to eq. (2.8) can become large, attaining a finite maximum value

$$e^{-4A_m} \simeq e^{8\pi K/3Mg_s}$$
 (2.15)

We define a throat as a region where the relative redshift,  $\Omega$ , is particularly large compared to other points on the manifold. Since  $\Omega$  is given by the ratio of  $e^A$  at the two points in question, and because the warp factor in the bulk is  $e^A \simeq c^{-1/4}$ , eq. (2.15) implies a maximum redshift

$$\Omega_m = \frac{(e^{-4A_m} + c)^{-1/4}}{c^{-1/4}} = \left(1 + \frac{e^{-4A_m}}{c}\right)^{-1/4},\tag{2.16}$$

relative to the bulk. Note that this depends on the volume of the compactification through its dependence on  $c \propto \mathcal{V}^{2/3}$ . We have a strongly warped throat if

$$c \ll e^{-4A_m} \tag{2.17}$$

and this redshift factor is large:  $\Omega_m \simeq e^{A_m} c^{1/4} \propto e^{A_m} \mathcal{V}^{1/6}$ . The condition (2.17) moreover implies that the geometry (2.6) in the throat region is largely independent of c and so also of the overall volume of the compactification. If, on the other hand,

$$c \gg e^{-4A_m} \tag{2.18}$$

then the relative redshift (2.16) tends to unity and the geometry is that of a usual Calabi-Yau compactification.<sup>3</sup>

Fluxes generically introduce masses for complex structure moduli and the dilaton [2], regardless of whether or not they break supersymmetry, and we pause here to describe how these scale with  $\mathcal{V}$  and  $e^{-4A_m}$  as a warm-up for our later discussion of the gravitino. The mass spectrum of the excitations of the dilaton (which can be thought of as representative of modes that acquire flux-induced masses) was studied in detail in [16], and is obtained by linearizing the equations of motion [15] for the dilaton about the backgrounds of [2], as described in previous sections. Linearizing the equation of the dilaton for an excitation with four-dimensional mass m and wave-function in the internal direction  $\tau(y)$ , one obtains [15] in this way<sup>4</sup>

$$\lambda e^{4A}\tilde{\nabla}^2 \tau(y) + m^2 \tau(y) = \frac{g_s}{12} \lambda e^{8A} G_{mnp} \bar{G}^{\widetilde{mnp}} \tau(y). \tag{2.19}$$

<sup>&</sup>lt;sup>3</sup>This is in keeping with the fact that fluxes are  $\alpha'$  effects, and so all flux-induced effects should disappear for sufficiently large volume. This argument also applies to settings other than type IIB.

<sup>&</sup>lt;sup>4</sup>In obtaining (2.19) we have ignored mixing with the fluctuations of the three form and metric. We shall use the equation only to give a qualitative explanation of the results of [16], a purpose for which these fluctuations are not important.

Notice that the 'mass' term generated by the flux,

$$\frac{g_s}{12}\lambda e^{8A}G_{mnp}\bar{G}^{\widetilde{mnp}}, \qquad (2.20)$$

is not a constant but varies non-trivially over the internal manifold.

In the bulk region, the metric  $\tilde{g}_{mn}$  is of order unity, hence  $G_{mnp}\bar{G}^{\widetilde{mnp}}$  is of order  $n_f^2$ , where  $n_f$  is the flux quantum which measures the amplitude of G. Using  $e^{-4A} \sim c$  for the bulk, we find

$$g_s \lambda e^{8A} G_{mnp} \bar{G}^{\widetilde{mnp}} \sim \frac{\lambda n_f^2}{\mathcal{V}^{4/3}},$$
 (2.21)

leading to [15, 16]

$$m_{\tau} \sim \frac{n_f \sqrt{\lambda}}{\mathcal{V}^{2/3}}$$
 (2.22)

This should be compared with the scaling of a generic 10D or KK mass, such as those implied by  $^5$  eq. (2.7):

$$M_{10} \sim m_{10} \sqrt{\lambda} \left( e^{-4A_m} + c \right)^{-1/4} \sim \frac{\sqrt{\lambda}}{\mathcal{V}^{1/6}} m_{10}$$
  
and  $M_{KK} \sim \sqrt{\lambda} \left( e^{-4A_m} + c \right)^{-1/2} \frac{1}{L} \sim \frac{\sqrt{\lambda}}{\mathcal{V}^{1/3}} \frac{1}{L}$ , (2.23)

where L is a characteristic length scale measured using the metric  $\tilde{g}_{mn}$ , and the last approximate equalities in both cases specialize to the bulk and use  $c \propto \mathcal{V}^{2/3}$ .

In the throat region, by contrast, the underlying Calabi-Yau metric  $\tilde{g}_{mn}$  has a shrinking three cycle, and the presence of the fluxes makes the internal manifold (with metric  $e^{-2A}\tilde{g}_{mn}$ ) have a characteristic length scale in this region of the order of  $\sqrt{n_f'}$  in string units, where  $n_f'$  is an integer which quantizes the amplitude of fluxes threading the cycles in the throat region.<sup>6</sup> Provided the wave-function of the mode of interest is localized in this throat region, we therefore expect  $G_{mnp}\bar{G}^{mnp}\sim 1/n_f'$  (and so  $G_{mnp}\bar{G}^{mnp}\sim e^{-6A_m}/n_f'$ , where each factor of  $\tilde{g}^{mn}$  contributes a power of  $e^{-2A_m}/n_f'$ ). Hence at the bottom of the throat we find

$$g_s \lambda e^{8A} G_{mnp} \bar{G}^{\widetilde{mnp}} \sim \frac{\lambda e^{2A_m}}{n_f'},$$
 (2.24)

leading to a KK mass of order [16]

$$m_{\tau} \sim e^{A_m} \sqrt{\frac{\lambda}{n_f'}} \,.$$
 (2.25)

For comparison, eqs. (2.23) give the following estimates for 10D masses and KK mode energies in the strongly warped region

$$M_{10}^{w} \sim \sqrt{\lambda} e^{A_m} m_{10}$$
and  $M_{KK}^{w} \sim \frac{\sqrt{\lambda} e^{2A_m}}{L} \sim \frac{\sqrt{\lambda} e^{A_m}}{\rho}$ , (2.26)

<sup>&</sup>lt;sup>5</sup>Recall that  $\lambda \sim \mathcal{V}^{-2/3}$ ,  $L \sim 1$  in our conventions and in the bulk  $m_{10} \sim \mathcal{V}^{-1/2}$ .

<sup>&</sup>lt;sup>6</sup>For instance in the infrared end of the KS throat  $n'_f \sim M$ , the flux threading the  $S_3$  of the conifold.

where we again use that the cycle size measured by the metric  $\tilde{g}_{mn}$  deep inside a throat scales with the warping as  $L \sim e^{A_m} \rho$ , where  $\rho$  is warping independent.

One expects the dilaton to localize in the infrared end of the throat region and acquire a mass of the order of (2.24) whenever it is energetically favorable to do so, i.e. when the volume is small enough so that (2.24) is less than (2.21). This yields the condition

$$\mathcal{V}^{2/3} \lesssim e^{-A_m},\tag{2.27}$$

which is equivalent to  $c \lesssim e^{-A_m}$ . Since  $c \sim \mathcal{V}^{2/3} \gg 1$ , eq. (2.17) ensures this condition is satisfied within a strongly warped throat.

The phenomenon of localization of massive modes should be fairly generic. As the volume of the compactification decreases, the redshift (2.16) becomes more and more prominent and one expects energetics to drive the wave-function of excitations into the throat region, ensuring the localized mode is continuously connected to the lowest mode in the regime (2.18). That is, as the volume of the compactification is decreased the wave-function of the dilaton continuously varies from being uniformly spread throughout the internal manifold to being highly localized in the throat. Furthermore, since the generic KK mass gap in the strongly-warped regime (2.17) is of the same order as the mass (2.25), all modes of the dilaton-axion should be integrated out of the effective field theory for the majority of 4D applications.

Our analysis of the gravitino (to follow in section 3) finds similar effects. In the presence of SUSY-breaking flux the wave-function of the lightest KK modes also localize into the throat region. In what follows we shall confine our discussion to this strongly-warped regime and examine its implications for SUSY breaking. But first we comment on various important energy scales and possible effective descriptions in this regime.

# 2.1.2 Energy scales and effective descriptions in the strongly warped regime

Let us be more explicit about scales in the strongly-warped situation, with  $c \lesssim e^{-A_m}$ . The relevant energy scales are: the four-dimensional Planck mass  $M_p$ , which is the basic scale in the Einstein frame and so convenient to set to unity. Notice that the bulk string scale is  $M_s \sim g_s \mathcal{V}^{-1/2}$ , and the bulk KK scale in the Einstein frame is  $M_{KK} \sim \mathcal{V}^{-2/3}$  in these units (recall eqs. (2.23) with  $\lambda \sim \mathcal{V}^{-2/3}$ ). Moreover, strong warping produces the warped string scale  $M_s^w \sim g_s e^{A_m} \mathcal{V}^{-1/3}$  and the warped Kaluza-Klein scale,  $M_{KK}^w \sim g_s e^{A_m} \mathcal{V}^{-1/3}/\rho \sim M_s^w/\rho$  (c.f. eqs. (2.26)) where  $\rho > 1$  is a characteristic length of the tip of the throat (in units of the string length). Notice that the volume dependence of  $M_{KK}^w$  and  $M_s^w$  is the same and they only differ somewhat by the factor  $\rho$ . This is an important difference with respect to the bulk quantities since this implies that the tower of string states and Kaluza-Klein states at the tip of the throat are not hierarchically different as  $\mathcal{V}$  gets large.

Based on this structure of scales we can distinguish the following energy regimes (and effective theories which describe them) in strongly warped models.

1.  $E < M_{KK}^w$ : This energy range is below the mass of moduli that acquire flux induced mass and all KK and string modes. The low-energy effective theory describing these

energies is necessarily a 4D field theory. It contains light degrees of freedom (like the Kähler moduli, which are massless until  $\alpha'$  corrections and non-perturbative effects are accounted for [2, 5]).

- 2.  $M_{KK}^w < E < M_s^w$ : This energy range (which can exist if  $\rho \gg 1$ ) is below the mass of all massive string states, warped or not, but contains a (finite) tower of those KK modes which are localized deep within the warped region. Because all string modes are massive, they can be integrated out leaving a low-energy effective theory which in this case is an explicitly higher-dimensional field theory. This effective theory is not the higher-dimensional field theory describing the full compact space. (If it were it would include states having masses larger than  $M_{KK}$ , which is larger than the assumed cutoff.) Instead it only probes the warped geometry, with cutoff at a point y = Y where the warp factor,  $e^{A(Y)}$ , is no longer sufficiently small. This energy scale represents both an UV cutoff (since it excludes KK and string states having energy higher than  $e^{A(Y)}/\mathcal{V}^{1/3}$ ) and an IR cutoff, since the condition y < Y gives the space a finite volume, providing an example of UV/IR mixing.
- 3.  $M_s^w < E < M_{KK}$ :<sup>7</sup> The effective theory for this energy range contains both massive KK and string modes, as well as non-perturbative excitations like branes and black holes, but again only those with strongly warped-suppressed spectra. As such, the low-energy effective theory must be a string theory again not the full string theory with which one starts, but rather a string theory which lives only within the cut-off volume of the warped geometry.
- 4.  $M_{KK} < E < M_s$ : This energy range includes only strongly warped string modes, but contains the KK modes of the higher-dimensional field theory. The low-energy effective theory in this case would appear to consist of the string theory localized to the warped region (as above), but coupled to the full set of supergravity modes which can propagate outside of the warped region. Such a theory is somewhat novel and it would be interesting to elucidate further its explicit description.
- 5.  $M_s < E$ : In this energy range the appropriate description is the full string theory, defined in the entire higher-dimensional geometry. It is believed that this theory can apply to arbitrarily high energies.

Cases 3 and 4 above may naïvely seem unusual inasmuch as they involve effective cut-off string theories, with the strings propagating in nontrivial, but cut-off, higher-dimensional background fields. They are indeed bona fide string theories since their cutoffs are much higher than the masses of the lightest (strongly warped) string states. Of course, in the light of AdS/CFT duality they seem less novel. One expects an alternate description of both regimes 2 and 3 above in terms of a cutoff gauge theory with the appropriate relevant deformation [24]. Moreover, one expects to be able to describe regime 4 in terms of the

<sup>&</sup>lt;sup>7</sup>Since we are assuming large warping we are taking  $M_s^w < M_{KK}$ . For small warping we can have  $M_{KK} < M_s^w$  and the regime  $M_{KK} < E < M_s^w$  is the standard 10D supergravity, as in the usual unwarped case.

supergravity of the bulk manifold coupled to a cutoff gauge theory description of the throat dynamics. This is a variant of the description of the effective field theory of the single brane RS scenario [8] as that of a cutoff conformal field theory coupled to four-dimensional gravity and other ultraviolet brane degrees of freedom [25, 26]. It would be interesting, but goes beyond the scope of this article, to further investigate properties of such theories, and to moreover understand the space of such possible theories and their possible deformations through excitation of stringy modes above the threshold  $M_s^w$ .

We now put aside such discussion and instead ask how supersymmetry breaking manifests itself in case 1.

#### 2.2 Supersymmetry breaking

We next turn to the relevance of these various scales for supersymmetry breaking.

# 2.2.1 Supersymmetry breaking scales and warping

Consider, then, a higher-dimensional compactification of a supersymmetric field (or string) theory. Although a generic compactification will break all of the supersymmetries of the higher-dimensional theory, we choose to focus here on compactifications — like those considered for Type IIB vacua in ref. [2], say — chosen to preserve, or to approximately preserve, one of these supersymmetries (from the 4D perspective). 'Approximate' conservation here means that the scale associated with splittings within the various 4D supersymmetry multiplets is sufficiently small, in a way made more precise below.

Any higher dimensional supersymmetry parameter,  $\varepsilon(x,y)$ , consists of many independent supersymmetries when viewed from the 4D perspective. Specifically, let a and  $\alpha$  be four- and six-dimensional spinor indices, respectively, so that the combination  $(a\alpha)$  serves as a ten-dimensional spinor index. Then  $\varepsilon$  can be expanded

$$\varepsilon^{a\alpha}(x,y) = \sum_{k} \epsilon_{k}^{a}(x) \, \eta_{k}^{\alpha}(y) \tag{2.28}$$

in terms of an appropriate basis of 6D spinors,  $\eta_k(y)$ , and each of the 4D spinors,  $\epsilon_k(x)$ , defines a separate local 4D supersymmetry transformation, gauged by the appropriate 4D component of the gravitino field,

$$\Psi_{\mu}^{a\alpha}(x,y) = \sum_{k} \psi_{\mu k}^{a}(x) \, \eta_{k}^{\alpha}(y) \,, \tag{2.29}$$

with  $\delta \psi_{\mu k} = D_{\mu} \epsilon_k + \dots$  under a supersymmetry transformation. We suppress spinor indices in the sequel.

If precisely one 4D supersymmetry is unbroken then by assumption one of the spinor modes,  $\eta_0$ , is an appropriate "Killing spinor," for which the corresponding gravitino mode  $\psi_{\mu 0}$  is precisely massless. Above this massless state will be a KK tower of massive 4D spin-3/2 states,  $\psi_{\mu k}$ , whose lightest elements we expect to have mass  $\sim M_{KK}$  in an unwarped environment, or  $\sim M_{KK}^w$  in a strongly warped environment. The unbroken supersymmetry ensures that the physics below all of these scales is captured by an effective 4D supergravity.

Imagine now breaking this last 4D supersymmetry, perhaps by turning on a nonzero flux, a configuration of branes or non-perturbative effects. Then all supersymmetries are broken, and in general there is no longer a uniquely defined gravitino which can be identified as 'the' 4D gravitino, to the extent that an approximate 4D description is possible. Two possible equivalent ways to define the 4D gravitino in this case are: (i) the gravitino having the lightest nonzero mass, since this gauges the 4D supersymmetry which is the least broken, and (ii) the gravitino which is adiabatically related to the massless gravitino as parameters are adjusted to restore an unbroken 4D supersymmetry.

In this section we argue that for strongly-warped systems the spin-3/2 state to which these lead has an extra-dimensional wave-function which is localized within the warped region. As a consequence it generically does not couple with  $M_p$ -suppressed couplings (unlike the massless 4D graviton), and so is unlikely to be approximately described by a simple 4D supergravity. In this case an alternative definition of the 4D gravitino is required, and experience with supersymmetric RS models [17, 18] suggests identifying the 4D gravitino as the state which couples to the strongly interacting CFT in the dual description of the strongly warped throat.<sup>8</sup> In this dual picture all of the lightest gravitino KK modes localized in the throat represent spin-3/2 resonances (similar to the massive KK graviton modes), and the much-more-massive, 'fundamental' gravitino is the state whose extra-dimensional wave-function most resembles that of the massless 4D graviton.

To study these issues in more detail, imagine turning on a supersymmetry breaking background field that is not itself confined to the strongly warped region, and so has an amplitude set by a scale,  $\tilde{\Lambda}$ , that is not warped. In what follows we take  $\tilde{\Lambda}$  to be bounded above by  $M_{KK}$ , as would be the case for flux-generated SUSY breaking. The quantity  $f = \tilde{\Lambda}/M_{KK} \lesssim 1$  is then a small (continuous or discrete) parameter, and one 4D supersymmetry is restored in the limit that  $f \to 0$ . As a result we know that when  $f \to 0$  all particles residing within the same 4D supermultiplet for this symmetry have precisely the same mass, and in particular one of the 4D KK gravitino modes is precisely massless. But for  $f \neq 0$  all 4D supersymmetries are broken and all KK mode masses are perturbed by an f-dependent amount which in general splits the masses of different particles residing within the same 4D supersymmetry multiplet.

We now ask: When  $f \neq 0$  how large are the splittings,  $\Delta m_k$ , within supermultiplets of the least broken 4D supersymmetry? And in particular, how massive is the lightest 4D gravitino? In the unwarped case, the answer for both questions would be  $m_{3/2} \sim \Delta m_k \sim f M_{KK}$ . In warped geometries masses of the same order are generically also expected for gravitino modes which are not localized within the strongly warped throat, such as for the explicit truncation of the 10D gravitino to the mode which is massless in the limit  $f \to 0$  [21],

$$\Psi_{\mu}(x,y) = \psi_{\mu 0}(x) \,\eta_0(y) \,. \tag{2.30}$$

We emphasize that such behaviour is expected only in the neighbourhood of the region of parameter space coresponding to  $f \to 0$ , away from this limit one expects the wavefunction of the gravitino to be modified and eventually to localize in the throat.

<sup>&</sup>lt;sup>8</sup>We thank T. Gherghetta and A. Pomarol for useful discussions on these points.

The next section, 2.2.2, shows that this same result is *not* true for the least broken 4D supersymmetry in warped geometries, whose gravitino is (by definition) the lightest gravitino KK state. To this end we generalize our earlier discussion for the dilaton to see how flux-induced gravitino KK masses scale with the parameters  $e^{A_m}$ , c and  $\mathcal{V}$  in warped geometries, and show that the lightest gravitino KK states are localized in the strongly warped regions, with energies that are characterized by the warped KK scale. In this case we argue that the mass of the least massive gravitino is warped to smaller values, being at most of order the warped KK scale if  $\tilde{\Lambda}$  is smaller than  $M_{KK}$ :

$$m_{3/2} \sim f' M_{KK}^w \,,$$
 (2.31)

provided  $f'M_{KK}^w \ll fM_{KK}$ . We quantify the size of the supersymmetry-breaking parameters f and  $f' \ll 1$  below.

#### 2.2.2 The gravitino mass

In this subsection we examine the gravitino equations of motion to obtain the criterion for the localization of the gravitino wavefunction and its mass in this regime, and to obtain estimates for the order of magnitude of the parameters f and f'. We shall follow the conventions in appendix B of [21] for the gravitino equations of motion, with  $\kappa = 1$ . First, we set our gamma matrix conventions. The 10 dimensional gamma matrices  $\Gamma^M$  satisfy the algebra

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN}. \tag{2.32}$$

For the metric (2.12), we take

$$\Gamma^{\mu} = \frac{e^{-A}}{\sqrt{\lambda}} \gamma^{\mu} \otimes 1 \quad \Gamma^{m} = e^{A} \gamma_{c} \otimes \tilde{\gamma}^{m}, \tag{2.33}$$

where

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu} \quad \{\tilde{\gamma}^m, \tilde{\gamma}^n\} = 2\tilde{g}^{mn}, \tag{2.34}$$

and  $\gamma_c = i\gamma_0\gamma_1\gamma_2\gamma_3$  is the four dimensional chirality matrix. The gravitino equation of motion is

$$\Gamma^{MNP}\hat{D}_N\Psi_P = -\frac{i}{2}\Gamma^P\Gamma^M\hat{\lambda}^*P_P - \frac{i}{48}\Gamma^{NPQ}\Gamma^M\hat{\lambda}G_{NPQ}^* + \mathcal{O}(\Psi^3)$$
 (2.35)

where  $\hat{\lambda}$  is the dilatino, and  $P_M$  is the field strength of the dilaton

$$P_M = \frac{1}{1 - BB^*} \partial_M B, \qquad (2.36)$$

with

$$B = \frac{1+i\tau}{1-i\tau} \ . \tag{2.37}$$

The supercovariant derivative acting on the gravitino is given by

$$\hat{D}_N \Psi_P = D_N \Psi_P - R_P \Psi_N - S_P \Psi_N^* \tag{2.38}$$

with

$$R_M = \frac{i}{480} (\Gamma^{M_1...M_5} F_{M_1...M_5}) \Gamma_M \tag{2.39}$$

and

$$S_M = \frac{1}{96} (\Gamma_M^{\ NPQ} G_{NPQ} - 9\Gamma^{NP} G_{MNP}) \ . \tag{2.40}$$

The complicated nature of the equations makes it difficult to find an explicit solution corresponding to the massive 4D gravitino. In fact, it is not consistent to excite just the fields  $\Psi^{\mu}$ ; as in the case of dilaton and complex structure moduli [15], one finds mixing between the various 10D supergravity modes while carrying out the KK reduction. Given the intractable nature of the equations of motion, we shall use energetic arguments to determine the condition for localization of the gravitino motivated by the observations (2.20), (2.27) for the dilaton in section 2.1.1.

We take the four dimensional gravitino  $\psi_{\mu}(x)$ , to be embedded in the ten dimensional gravitino as

$$\Psi_{\mu}(x,y) = \psi_{\mu}(x) \otimes \eta(y) \tag{2.41}$$

where  $\eta(y)$  is the wavefunction of the gravitino in the extra dimensions. Thus, components of the gravitino equation of motion in the non-compact direction have the structure

$$\frac{e^{-3A}}{\lambda^{3/2}} \gamma^{\mu\nu\rho} \partial_{\nu} \psi_{\rho} \otimes \eta(y) + \frac{1}{24\lambda} \gamma^{\mu\nu} \gamma_{c} \psi_{\nu}^{*} \otimes e^{A} G_{mnp} \gamma^{\widetilde{mnp}} \eta^{*} + \dots = 0, \qquad (2.42)$$

where we have not explicitly written the contributions of the terms containing derivatives of  $\eta(y)$  and other contributions neglected due to the specific form of our ansatz. The term

$$\gamma^{\mu\nu\rho}\partial_{\nu}\psi_{\rho} \tag{2.43}$$

is the standard kinetic term for a spin 3/2 field in four dimensions. One expects the mass to be of the order of the relative strength of the kinetic term and the term involving fluxes in (2.42). We assume that the terms not written explicitly in (2.42) scale in a similar way as a function of the warping and Calabi-Yau volume as do the terms that are shown, justifying their neglect in obtaining an estimate for the mass. This should be reasonable since all such terms must adjust so that the equations can be satisfied, and appears confirmed by a crude analysis of their effects in the equations of motion.

Comparing the kinetic and potential terms in (2.42) we find that the flux-induced mass term for the gravitino varies across the internal manifold, and scales like

$$\sqrt{\lambda}e^{4A}G_{mnp}\gamma^{\widetilde{mnp}}.$$
 (2.44)

Keeping in mind that the gamma matrices scale like the "square root of the inverse metric," and comparing with (2.20), we see that the mass term has the same dependence on the warp factor and the fluxes as the mass term for the dilation defined in (2.20). Thus, we find for weak warping  $(c \geq e^{-A_m})$  a flux-induced gravitino mass of order

$$m_{3/2} \propto \frac{\vartheta}{\mathcal{V}},$$
 (2.45)

where  $\vartheta$  is the strength of supersymmetry breaking fluxes. In terms of the definition  $f \sim m_{3/2}/M_{KK}$  we therefore have

$$f \sim \frac{\vartheta}{\mathcal{V}^{1/3}} \,. \tag{2.46}$$

Similarly, for strong warping  $(c \lesssim e^{-A_m})$  the lightest gravitino wave-function localizes, giving a mass of order (where  $\lambda \sim \mathcal{V}^{-2/3}$ )

$$m_{3/2} \propto \vartheta' e^{A_m} \sqrt{\frac{\lambda}{n_f'}}$$
, (2.47)

and so  $f' = m_{3/2}/M_{KK}^w$  is

$$f' \sim \frac{\vartheta' \rho}{\sqrt{n_f'}}$$
, (2.48)

where  $\vartheta'$  describes the relative strength of the SUSY breaking and supersymmetric fluxes that thread cycles that are localized within the throat.

To summarize, the above arguments provide two estimates for the size of KK gravitino masses arising from supersymmetry-breaking fluxes in warped geometries. In particular we expect those KK modes which are not localized in the warped areas to generically have masses which are of order  $\Delta M \equiv f M_{KK} \sim \vartheta/\mathcal{V}$ . By contrast, for strongly warped throats (i.e. those for which  $\mathcal{V}^{2/3} \ll e^{-A_m}$ ) some gravitino modes can lower their masses to  $\Delta m \equiv f' M_{KK}^w \sim \vartheta' e^{A_m} / \sqrt{n_f'} \, \mathcal{V}^{1/3}$  by localizing within the strongly warped areas.

### 2.2.3 The supersymmetric limit

We include the factors  $\vartheta$  and  $\vartheta'$  in the above expressions in order to capture the fact that fluxes can, but need not, break supersymmetry. Indeed, completely supersymmetric warped geometries with fluxes exist, and this limit is captured in the above expressions for  $m_{3/2}$  through the limit  $\vartheta \to 0$ . Notice also that the parameters f and f' can be made systematically small, either by taking small  $\vartheta, \vartheta'$  or large  $\mathcal V$  or  $n_f'$ . Being able to ensure small f and f' is important in the next section, where we discuss the low-energy 4D effective theory, and ask whether the mass of the lightest gravitino is hierarchically separated from the mass of other excitations.

Before examining the implications of these scales for the low-energy effective 4D theory, we first pause to argue that it is the lowest energy KK mode which is adiabatically related to the massless gravitino in the supersymmetric limit, even though its localization into the throat ensures that its extra-dimensional wave-function differs from the direct supersymmetric truncation, eq. (2.30). At first sight this is a surprising claim, particularly if the supersymmetry breaking parameter f is small, because the mode (2.30) represents the massless state when  $f \to 0$ . And in perturbation theory it usually suffices to use unperturbed eigenfunctions,  $\psi_0$ , in order to find the leading-order correction to the corresponding eigenvalues,  $\delta E \sim (\psi_0, H_{\rm int}\psi_0)$ , and so one might expect estimates based on the supersymmetric gravitino wave-function to therefore capture the leading contributions to the lightest nonzero gravitino mass once supersymmetry breaks.

Yet we have seen that in the strongly warped regime the wave-function of the lightest gravitino KK mode in the presence of supersymmetry breaking fluxes is localized in the highly warped region and so differs significantly from the Killing spinor on the Calabi Yau. This localization is responsible for the lowering of its mass to the warped KK scale, and is not captured by an expression like  $\delta E \sim (\psi_0, H_{\rm int}\psi_0)$  since this does not correct the wave-function. The key point is that for warped geometries one must use degenerate perturbation theory, because there are many unperturbed (supersymmetric) states having warped masses which can be much smaller than the typical matrix element of the perturbation,  $fM_{KK}$ . Because of this degeneracy the system can minimize its energy in the presence of the supersymmetry-breaking perturbation by choosing an appropriate linear combination of states, which in the present instance are the states localized within the warped throat.

If we imagine adiabatically turning on an infinitesimal supersymmetry-breaking perturbation (or deepening the warping of a throat), we expect non-degenerate perturbation theory to apply so long as  $fM_{KK} \ll f'M_{KK}^w$ , and in this limit the wave-function of the lightest gravitino KK mode should remain very close to the supersymmetric state, eq. (2.30). However, once the supersymmetry-breaking flux is large enough that  $fM_{KK}$  becomes larger than  $f'M_{KK}^w$  it becomes energetically favourable to concentrate into the throat, and so as  $\vartheta$  and  $\vartheta'$  are increased the lightest gravitino state continuously evolves into a localized state with a warped flux-induced KK mass. In this sense the evolution of the gravitino wave-function with increasing warping resembles the more familiar process of the repulsion of atomic energy levels or the resonant oscillations of neutrinos in matter. Further discussion of the nature of the KK wave-functions in highly warped geometries and the use of degenerate perturbation theory in this context is given in the appendix, where a toy example is analyzed in more detail.

#### 2.3 Criteria for supersymmetric low-energy actions

We have discussed, in section 2.1.2, when the low-energy action of a system should be 4-dimensional or higher dimensional, and how the existence of strongly warped throats complicates the procedures which are used in unwarped situations. In this section we similarly review the criteria for when the low-energy limit of a higher-dimensional supergravity (or string) theory should be described by a (possibly spontaneously broken) supersymmetric, or explicitly non-supersymmetric effective theory.

From the 4D point of view, higher dimensional supergravity theories broken by bulk flux fields are special cases of 'hidden sector' models (for a review see [27]). In these models there is a collection of low-energy fields,  $\ell^a$ , of physical interest (describing, say, Standard Model particles). These are assumed to be coupled to a more generic set of fields,  $h^m$ , whose dynamics somehow breaks supersymmetry. Although supersymmetry is badly broken in the 'hidden' sector described by  $h^m$  (with typical supermultiplet mass splittings of order  $\Delta M$ ), the weak  $h-\ell$  couplings ensure this is only weakly transmitted to the 'light' sector described by  $\ell^a$  (whose supersymmetry breaking splittings are  $\Delta m \ll \Delta M$ ). In the best-case scenario these couplings are gravitational in strength. Notice that there is no

<sup>&</sup>lt;sup>9</sup>We thank Henry Tve for comments on this point.

requirement that the hidden-sector fields be light, although there is typically one state in this sector, the goldstone fermion, <sup>10</sup> which is much lighter than the others.

The form of the low-energy effective field theory which describes this kind of situation below a UV cutoff,  $\Lambda$ , depends in an important way on the relative size of  $\Lambda$ ,  $\Delta M$  and  $\Delta m$ , as follows.

- 1.  $\Delta M \ll \Lambda$ : If the cutoff is larger than all supersymmetry-breaking mass splittings then the field content of the low-energy theory can be grouped into supermultiplets. In this case the low-energy theory is itself described by a supergravity, even though it may include some of the hidden-sector fields. Any spontaneous supersymmetry breaking within the full theory can be understood within the effective theory as spontaneous supersymmetry breaking due to the appearance of a SUSY-breaking v.e.v. purely within the effective theory.
- 2.  $\Lambda \ll \Delta m$ : If the cutoff is well below the smallest SUSY-breaking scale, then a generic supermultiplet has some elements which are heavier than  $\Lambda$  and so are integrated out, while others are lighter than  $\Lambda$  and so remain in the low-energy theory. In this case the field content of the low energy theory cannot be organized into supermultiplets (it might contain just fermions with no bosons, for example), and so supersymmetry must be nonlinearly realized [28]. (Notice that for a gauge theory a nonlinearly-realized spontaneous breaking is operationally indistinguishable from explicit breaking within the low-energy theory below the breaking scale [29], and so in this case the effective theory can be an arbitrary non-supersymmetric field theory.)
- 3.  $\Delta m \ll \Lambda \ll \Delta M$ : If the cutoff lies between the two splitting scales, then there generically are supermultiplets in the hidden sector (split by  $\Delta M$ ) for which some particles are heavier than  $\Lambda$  and so are integrated out, while others are lighter than  $\Lambda$  and so remain in the low-energy theory. This is true in particular for the multiplet which contains the goldstone fermion in the hidden sector. In this case supersymmetry is generically badly broken in the effective theory. What distinguishes this case from case 2 above, is that supersymmetry breaking is much smaller than  $\Lambda$  within the light sector, which therefore has the field content to fill out complete supermultiplets. As a result, provided we restrict our attention only to light-sector observables the breaking of supersymmetry in this sector can be described as a supergravity coupled to a collection of soft-breaking terms<sup>11</sup> [30, 31] which encode the couplings to supersymmetry breaking in the hidden sector.

For extra-dimensional supergravity without warping, the low-energy theory of interest is often defined to cover energies smaller than the KK scale,  $M_{KK}$ , and so consists of the effective 4D interactions of the various KK zero modes. In this case we can regard the

<sup>&</sup>lt;sup>10</sup>Or, more precisely, the massive gravitino which 'eats' it.

<sup>&</sup>lt;sup>11</sup>By 'soft breaking' we do not here mean terms which do not generate quadratic divergences, but instead have in mind the more general usage, spelled out in more detail below, in which supersymmetry breaking arises through terms obtained by replacing hidden-sector auxiliary fields by their SUSY-breaking expectation values.

hidden sector to consist of the massive KK modes and the light sector to consist of the low-energy KK zero modes. If supersymmetry is broken by extra-dimensional physics, such as by fluxes, then the low-energy 4D theory below the KK scale in general need not be supersymmetric. If, however, the mass splitting,  $\Delta m \sim f M_{KK}$ , among the KK zero modes satisfies  $\Delta m \ll M_{KK}$  (and so  $f \ll 1$ ), the above discussion shows that the low-energy theory can be an effective 4D supergravity (possibly coupled to soft-breaking terms which capture the effects of integrating out parts of badly-split KK supermultiplets).

In the case with strong warping, we can take the light sector to consist of those modes which are localized within the strongly warped region, whose masses are typically of order  $M_{KK}^w$  or smaller. The hidden sector consists of those fields which are not so localized. We have seen that supersymmetry-breaking splittings in the light sector are of order  $\Delta m = f' M_{KK}^w$  while those in the hidden sector can be much larger, of order  $\Delta M = f M_{KK}$ . (Figure 1 sketches some of the relevant scales for the light modes in the two cases,  $\Delta m \ll \Delta M \ll M_{KK}^w$  and  $\Delta m \ll M_{KK}^w \ll \Delta M$ .) A complicating feature in this case is that a 4D description is not valid at all unless a hierarchy exists between the states of interest and the generic warped KK scale  $\Lambda \ll M_{KK}^w$ , and this generically need not be the case if all of the light gravitino states are split by  $\Delta m \sim f' M_{KK}^w$ , which is of the same order as the lightest gravitino mass.

As a result, in the strongly warped case the absence of a clean hierarchy between the scale of supersymmetry breaking and the onset of the extra-dimensional description indicates we should generically not expect to obtain a low-energy description consisting purely of a 4D supergravity, even supplemented by soft supersymmetry breaking terms. Conversely, if a single gravitino were much lighter than the others, its interactions would be well described by a 4D supergravity, and in particular would necessarily be suppressed by the 4D Planck scale,  $M_p$ , as are those of its superpartner, the massless 4D graviton. However, the interactions of localized gravitino states in strongly warped regions are generically suppressed at low energies only by a warped scale, such as  $M_{KK}^w$ , and so are generically much stronger than are those of the massless 4D graviton.

These considerations suggest instead formulating the effective 4D theory in terms of the dual variables, in terms of which the semiclassical geometrical degrees of freedom in the throat are described by some sort of strongly interacting conformal field theory (CFT). In this picture the stronger interactions of the tower of warped KK gravitons and gravitini are regarded as expressing the relatively strong residual interactions amongst resonant spin-3/2 and spin-2 bound states of the CFT constituents, with the corresponding supersymmetries being emergent symmetries associated with the strong CFT interactions.

The presence of a strongly-coupled CFT makes the criteria for describing this dual system in terms of a 4D supergravity more complicated to formulate. However a generic situation for which further progress is possible is in the case where we choose not to directly measure properties of the CFT and instead integrate it out and ask for its implications on other low-energy states (like the 4D graviton, bulk moduli, and so on). Provided the couplings between these low-energy modes and the CFT are sufficiently weak the CFT can simply act as a hidden sector, whose implications might be captured by a (possibly softly broken) effective 4D theory. Because the 4D gravitino which would be relevant to

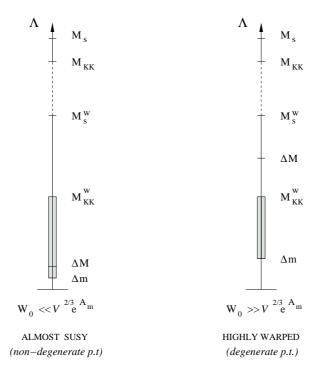


Figure 1: Relevant scales for the localized modes at the infrared bottom of a throat. In the case  $W_0 \ll \mathcal{V}^{2/3}e^{A_{min}}$ , the effects of the fluxes are almost negligible and the system is nearly supersymmetric. For  $\mathcal{V}^{2/3}e^{A_{min}}\gg W_0$ , the effects of the fluxes however become important. The masses of the unperturbed states  $M_{KK}^w$  are much smaller than the flux scale and non-degenerate perturbation theory no longer applies to this case. The shaded region shows the range of energies at which an ultraviolet cutoff gives rise to a 4D supergravity description for the light modes.

such a theory would have couplings similar to the 4D graviton, it could be expected to be a bulk state whose mass is of order  $fM_{KK}$ , and so considerably heavier than the warped KK scales. This more complicated situation is in the spirit taken by much of the model building within supersymmetric 5D RS models [17, 18].

#### 2.3.1 Effective 4D supersymmetry

Suppose first we consider case 1, for which all supersymmetry breaking scales are small compared with the warped KK scale. (We might imagine this is arranged, for instance, by appropriately choosing the parameter  $\vartheta$ .) In this case we expect the low energy limit to be given by a 4D effective supergravity, described, as usual, by a Kähler potential,  $K(\ell, \ell^*)$ , a superpotential,  $W(\ell)$ , and a gauge kinetic function,  $F_{AB}(\ell)$ . For instance, leading order calculations in Type IIB flux compactifications give

$$K = -2\ln \mathcal{V}, \quad \text{and} \quad W = W_0, \tag{2.49}$$

where  $\mathcal{V}$  is the volume of the underlying Calabi-Yau space expressed as a function of its holomorphic moduli, and  $W_0$  is the Gukov-Vafa-Witten superpotential [32], regarded as a

function of the holomorphic complex structure moduli.

If all supersymmetries are broken by the fluxes in the underlying compactification, N=1 4D SUSY will be spontaneously broken in the 4D effective theory, leading to a mass for the 4D gravitino of the form

$$m_{3/2} = e^{K/2} |W| = \frac{|W_0|}{\mathcal{V}},$$
 (2.50)

in the absence of strong warping. This uses the lowest-order results  $K = -2 \ln \mathcal{V}$  and  $W = W_0$ . We see that the freedom to dial fluxes to small values in the microscopic theory to obtain small supersymmetry breaking effects is described in the low-energy theory by the freedom to choose very small values for  $W_0$ . Also, comparing this with (2.45) we find that in the regime that there is a four dimensional effective description,  $\vartheta \sim |W_0|$ .

As discussed above, in the case of strong warping it is not generic that the low-energy theory is well described by a 4D supergravity at all. However, it can happen that a 4D supergravity can apply, even if there exist strongly warped regions. One such a case arises in the supersymmetric limit, for which there is always a clear hierarchy between the massless gravitino and the warped or unwarped KK scales. Suppose the low-energy matter sector in this case consists of a strongly warped throat. The theory then contains warped states, whose low-energy interactions with the supergravity sector can be of interest.

In such a case we would expect an effective 4D supergravity which must know about the warped scale, and we now ask how this might be encoded into the low-energy theory. Since the influence of fluxes in general arise at higher order in  $\alpha'$ , the modifications to the low-energy supergravity which encode this warping might be expected to arise as  $\alpha'$  corrections, and so in particular arise as modifications to K. (In principle the explicit form for this warping dependence could be captured by matching to the underlying Calabi-Yau dynamics, such as was done for 5D supersymmetric RS models in ref. [20], but calculations are hampered in the 10D case by not knowing the explicit form for the underlying geometry in this case.)

Part of the answer of how warping can enter K to describe states that are localized in the throat is given by adding  $2A_m$  to K, since this provides an appropriate overall scaling down of all masses (with fixed 4D Planck mass),

$$K(\ell, \ell^*) = 2A_m + \check{K}(\ell, \ell^*),$$
 (2.51)

with the holomorphic functions W and  $F_{AB}$  unchanged. Such a constant piece does not contribute at all to the Kähler metric,  $\partial_a\partial_{\overline{a}}K$ , or to the covariant derivative,  $D_aW=\partial_aW+W\partial_aK$ , but does have the effect of scaling the scalar potential by an overall factor:  $U=e^{2A_m}\check{U}$ , where  $\check{U}$  is the scalar potential computed using the Kähler potential  $\check{K}$ . In particular, the entire scalar mass matrix gets scaled by a corresponding amount,  $m^2=e^{2A_m}\check{m}^2$ . Fermion masses also get scaled down in an identical way, due to the ubiquitous factor of  $e^{G/2}$  to which they are proportional, with

$$G = K + \ln W + \ln W^*. (2.52)$$

The 4D gravitino does not share this warping in the supersymmetric limit because it is massless, by assumption. Such a constant contribution to K also appears as a leading

term in a large-volume expansion of the Kähler function identified for supersymmetric RS models [20].

#### 2.3.2 4D action with softly broken supersymmetry

We next turn to the case of more direct phenomenological interest, case 3, wherein we obtain our effective theory by integrating out a supersymmetry-breaking sector but keep the goldstone fermion whose mixing with the 4D gravitino gives it its mass. In this case we expect that if the SUSY-breaking sector couples sufficiently weakly to the observable sector, then its supersymmetry-breaking effects can be encoded in the low-energy theory by a suitable class of weakly-coupled soft-breaking interactions.

A warped example which is described by this type of soft breaking is obtained by perturbing our earlier supersymmetric picture of a supersymmetric throat. Suppose we now turn on a supersymmetry-breaking flux in the throat, but not by enough to localize the lightest gravitino KK mode. More precisely, suppose  $f'M_{KK}^w \gtrsim fM_{KK}$  even though  $M_{KK}^w \ll M_{KK}$ , as would be possible if  $\vartheta/\vartheta' \lesssim e^{A_m} \mathcal{V}^{2/3} \ll 1$  (and so, in particular, if  $\vartheta \to 0$ ). In this situation it does not energetically pay the lightest gravitino to localize, allowing it to have  $M_p$ -suppressed couplings.<sup>12</sup> Yet its mass is warped because this sets the scale of the SUSY-breaking physics in the warped throat.

In this case we expect the low-energy theory below the bulk KK scale to be a 4D supergravity coupled to a strongly interacting supersymmetric CFT describing the throat. Alternatively, integrating out the CFT can lead to a softly-broken 4D supergravity describing the remaining, observable, sector provided only that the couplings to the CFT are sufficiently weak. We now show that the generic warping of the resulting supersymmetry-breaking masses may also be accomplished by the same shift as for the supersymmetric case,  $K = 2A_m + \check{K}$ .

To show this we require a statement of what the resulting soft-breaking interactions might be. These have been enumerated in the literature [30, 31], under the assumption that there is a regime for which the full theory — both  $\ell^a$ 's and  $h^m$ 's — is given by a 4D N=1 supergravity, described by an appropriate Kähler potential, <sup>13</sup>

$$K = \hat{K}(h, h^*) + \tilde{K}_{a\overline{b}}(h, h^*)\ell^a\ell^{\overline{b}} + \frac{1}{2}Z_{ab}(h, h^*)\ell^a\ell^b + \cdots,$$
 (2.53)

as well as a superpotential, W, and gauge-kinetic function,  $F_{AB}$ . (Here  $\ell^{\overline{a}}$  denotes  $(\ell^a)^*$ .) In this case the soft-breaking quantities can be computed in terms of the assumed coupling

 $<sup>^{12}</sup>$ We could also entertain the possibility of  $f' \ll f$  and a light gravitino localized in the warped region. While this is an interesting scenario, an explicit calculation supporting the existence of such a regime is not available at present.

 $<sup>^{13}</sup>$ Although in the previous section the hidden fields h included moduli and matter fields living far from the throat, here they are understood to include only moduli which may survive at low energies (such as Kähler moduli).

functions and the SUSY-breaking hidden-sector auxiliary fields,

$$\mathcal{F}^m = e^{G/2} K^{m\overline{n}} \partial_{\overline{n}} G, \qquad (2.54)$$

using eq. (2.52). For instance, the resulting expressions for the scalar and gaugino masses are [31]

$$m_{a\overline{b}}^{2} = (m_{3/2}^{2} + V_{0})\tilde{K}_{a\overline{b}} - \mathcal{F}^{m}\mathcal{F}^{\overline{n}} \Big( \partial_{m}\partial_{\overline{n}}\tilde{K}_{a\overline{b}} - \tilde{K}^{c\overline{d}}\partial_{m}\tilde{K}_{a\overline{d}}\partial_{\overline{n}}\tilde{K}_{c\overline{b}} \Big)$$

$$M_{AB} = \frac{1}{2} \left[ (\operatorname{Re} F)^{-1} \right]_{AC} \mathcal{F}^{m}\partial_{m}F_{CB} , \qquad (2.55)$$

where as before the gravitino mass is  $m_{3/2} = e^{K/2}|W|$  and  $V_0$  is the value of the potential at its minimum.

We again expect the low-energy theory to know that all of the nonzero masses associated with localized states within the warped region are suppressed by a common factor  $e^{A_m}$ . As may be seen from the above formulae, this can be done if the Kähler potential contains an additive constant, which can be taken to be in the part describing the hidden sector

$$\hat{K}(h, h^*) = 2A_m + \mathcal{K}(h, h^*), \qquad (2.56)$$

for essentially the same reason as for the supersymmetric case considered above. Such a constant has the effect of scaling all of the supersymmetry breaking v.e.v.'s,  $\mathcal{F}^m$ , by a common factor of  $e^{A_m}$ , thereby ensuring that all masses are properly suppressed by the warp factor.

Similarly, using eqs. (2.46) in the expression for the mass of the 4D gravitino itself gives

$$m_{3/2} \sim e^{A_m} e^{\check{K}/2} W_0 \sim \frac{\vartheta e^{A_m}}{\mathcal{V}^{1/3}},$$
 (2.57)

This can be used as a guide to determine the Kähler potential in the regime of strong warping.

#### 3. SUSY breaking in the microscopic theory

In this section we give examples of how visible-sector supersymmetry-breaking masses might arise within a microscopic calculation within a warped environment. To this end we compute the warp-factor dependence of the masses which are induced for some brane moduli as a consequence of bulk fluxes.

#### 3.1 Flux-induced masses on a D3-brane

We consider a D3-brane filling the non-compact dimensions of a generic GKP vacuum [2]. The matter content of the theory in the world-volume of the brane will contain six real scalars  $Y^m$  parameterizing the position of the D3-brane in the transverse space. The dynamics of these scalars is described in terms of the corresponding Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions

$$S_3 = -|\mu_3| \int d^4x \ e^{-\phi} \sqrt{-\det(P[E])} + \mu_3 \int P[C_4] + \dots, \tag{3.1}$$

where  $\mu_3$  is the D3-brane charge, and P[E] denotes the pullback to the brane of the tensor  $E_{MN} = g_{MN} + B_{MN}$ . With brane coordinates  $(x^{\mu}, Y^m(x^{\mu}))$ , this can be written

$$P[E]_{\mu\nu} = E_{\mu\nu} + E_{mn}\partial_{\mu}Y^{m}\partial_{\nu}Y^{n} + \partial_{\mu}Y^{m}E_{m\nu} + \partial_{\nu}Y^{n}E_{\mu n}.$$
(3.2)

 $P[C_4]$  similarly denotes the pull-back of the RR 4-form potential. One expects additional terms in the CS piece due to the other RR fields, but these turn out to be irrelevant for the purpose of computing the scalar masses, as one may check. The metric in these expressions is the string-frame metric, which is related to (2.12) by an  $e^{\phi/2}$  scaling factor,

$$ds_{str}^{2} = e^{\phi/2} \left( \lambda e^{2A(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + e^{-2A(y)} \tilde{g}_{mn} dy^{m} dy^{n} \right),$$
 (3.3)

with  $\lambda$  defined in (2.13).

Assuming a constant background for the ten dimensional dilaton, as generically happens in the absence of D7 branes, one easily shows that

$$\sqrt{-\det(P[E])} = \lambda^2 e^{4A} e^{\phi} \left[ 1 + \frac{1}{2} \lambda^{-1} e^{-4A} \tilde{g}_{mn} \partial_{\mu} Y^m \partial^{\mu} Y^n \right]$$
(3.4)

where we keep only the terms relevant for the computation of the scalar masses and where the internal metric  $\tilde{g}_{mn}$  in (3.4) is evaluated at the position of the brane. From this we find

$$S_3 \simeq -\int d^4x \left[ |\mu_3| \frac{\lambda}{2} \tilde{g}_{mn} \partial_\mu Y^m \partial^\mu Y^n + |\mu_3| \lambda^2 e^{4A} - \mu_3 C_{0123} \right] . \tag{3.5}$$

Thus the potential for a D3 position  $Y^m$  is given by

$$V = \lambda e^{4A} - \frac{1}{\lambda} C_{0123} . {3.6}$$

This can be expanded around the minimum value  $e^{4A_m}$  to give the mass matrix,

$$V \simeq \lambda e^{4A_m} + \frac{\partial_m \partial_n V}{2} Y^m Y^n \ . \tag{3.7}$$

The trace of the mass matrix can be computed (see e.g. [35, 36]) using the supergravity equations [2]:

$$\tilde{g}^{mn}\partial_m\partial_n V = \lambda \tilde{\nabla}^2 \left( e^{4A} - \frac{1}{\lambda^2} C_{0123} \right) \simeq \lambda \frac{e^{8A}}{24 \operatorname{Im}\tau} (G_3 + i\tilde{*}_6 G_3)_{mnp} (\bar{G}_3 - i\tilde{*}_6 \bar{G}_3)^{\widetilde{mnp}} . \tag{3.8}$$

The fact that the scalar masses vanish for imaginary self dual fluxes can be traced back to the no-scale structure of these Type IIB vacua [21, 37]. However, imaginary anti-self dual components for the 3-form flux can be thought of as being induced by the backreaction of effects which break the no-scale structure. Comparing (3.8) to (2.24), we find that the flux-induced scalar masses evaluated in a warped throat are of the same size as both the bulk moduli masses discussed there and the gravitino mass (2.47),

$$m_Y \sim e^{A_m} \sqrt{\lambda} \ .$$
 (3.9)

Notice that, with very little effort, the computation can be extended to the case of the scalar masses for a  $\overline{D3}$  brane, as its action differs only in the sign of the last term in (3.5). Thus, the expression (3.9) remains valid for an antibrane, but now the left hand side of (3.8) is proportional to the imaginary self dual components of the fluxes [36].

#### 3.2 Flux-induced masses on a D7 brane

For D7 branes one can proceed as in the previous section. However, now the D7 brane wraps a 4-cycle  $\Sigma$  in the extra dimensions. We consider a trivial normal bundle for the embedding of  $\Sigma$  in the compact manifold, so there are just two geometric moduli  $Y^i$  in the four dimensional theory parameterizing the position of the D7-brane. We use 4D indices  $\alpha, \beta, \ldots$  for the compact dimensions within the brane volume; and 2D indices  $i, j, \ldots$  for the compact dimensions transverse to the D7-brane. If we imagine the D7 to wrap directions 4,5,6,7 of the compact space so that 8,9 denote the transverse directions, then  $\alpha, \beta, \ldots = 4, \ldots, 7$  and  $i, j, \ldots = 8, 9$ .

For simplicity, we only consider the reduction of the DBI piece of the action, which for a D7-brane reads,

$$S_7 = -|\mu_7| \int_{\mathbb{R}^4 \times \Sigma} d^8 \xi \ e^{-\phi} \sqrt{-\det(P[E])} \ .$$
 (3.10)

Notice that, as for D3-branes, one expects also contributions from the CS action, cancelling part of the mass terms in the DBI piece.<sup>14</sup> However, for the purpose of computing the warp suppression of the scalar masses, it suffices to analyze the DBI contributions.

The pull-back (3.2) is then given by

$$P[E]_{\mu\nu} = e^{2A} \lambda e^{\phi/2} \eta_{\mu\nu} + e^{-2A} e^{\phi/2} \tilde{g}_{ij} \partial_{\mu} Y^{i} \partial_{\nu} Y^{j}$$
  

$$P[E]_{\alpha\beta} = e^{-2A} e^{\phi/2} \tilde{g}_{\alpha\beta} + B_{\alpha\beta} ,$$
(3.11)

where we restrict ourselves to the case  $B_{\mu\nu} = 0$  and  $\partial_{\alpha}Y^{i} = 0$ . The two fields  $Y^{i}(x)$  denote low-energy fluctuations in the transverse position,  $y^{i} = Y^{i}(x)$ , of the D7 brane.

Because of its block-diagonal structure, the determinant of P[E] becomes

$$\det(P[E]) = (e^{2A}\lambda e^{\phi/2})^4 \det\left[\eta_{\mu\nu} + e^{-4A}\lambda^{-1}\tilde{g}_{ij}\partial_{\mu}Y^i\partial_{\nu}Y^j\right]$$

$$\times (e^{-2A}e^{\phi/2})^4 \det\left[\tilde{g}_{\alpha\beta} + e^{2A}e^{-\phi/2}B_{\alpha\beta}\right],$$
(3.12)

which, when expanded in powers of the fluctuations gives

$$\sqrt{-\det(P[E])} = \lambda^2 e^{2\phi} \sqrt{\tilde{g}_4} \left( 1 + \frac{1}{2} e^{-4A} \lambda^{-1} \tilde{g}_{ij} \partial_{\mu} Y^i \partial^{\mu} Y^j + \cdots \right) 
\times \left( 1 + \frac{1}{4} e^{4A} e^{-\phi} B_{\alpha\beta} B^{\widetilde{\alpha}\widetilde{\beta}} + \cdots \right), \tag{3.13}$$

where we denote det  $\tilde{g}_{\alpha\beta}$ , the determinant of the pullback on the four cycle, by  $\tilde{g}_4$ . We can use this in the DBI action, working to quadratic order in the fluctuations; we assume that the flux  $B_{\alpha\beta}$  is constant over the four cycle, and hence it does not induce Freed-Witten anomalies [42] in the worldvolume of the brane. This gives

$$S_{7} = -|\mu_{7}| \int_{\mathbb{R}^{4} \times \Sigma} d^{4}x \, d^{4}y \, \sqrt{\tilde{g}_{4}} \, e^{\phi} \left( \lambda^{2} + \frac{1}{2} e^{-4A} \lambda \tilde{g}_{ij} \partial_{\mu} Y^{i} \partial^{\mu} Y^{j} + \frac{1}{4} e^{4A} \lambda^{2} e^{-\phi} B_{\alpha\beta} B^{\widetilde{\alpha\beta}} + \cdots \right)$$

$$= -|\mu_{7}| \int d^{4}x \, \left( V + \frac{1}{2} G_{ij} \partial_{\mu} Y^{i} \partial^{\mu} Y^{j} \right)$$

$$(3.14)$$

<sup>&</sup>lt;sup>14</sup>See [38] for further details.

where the potential and kinetic terms are given by

$$V = \lambda^2 \int_{\Sigma} d^4 y \sqrt{\tilde{g}_4} e^{\phi} \left[ 1 + \frac{1}{4} e^{-\phi} e^{4A} B_{\alpha\beta} B^{\widetilde{\alpha}\widetilde{\beta}} \right], \quad G_{ij} = \lambda \int_{\Sigma} d^4 y \sqrt{\tilde{g}_4} e^{-4A} e^{\phi} \tilde{g}_{ij} \quad . \tag{3.15}$$

We are now in a position to see why the presence of a nonzero background flux,  $H_{mnp} \neq 0$ , can generate a mass for the brane modulus  $Y^i$ . Notice that since  $dB = H_3$ , if the normal to  $\Sigma$  has components along the cycle supporting  $H_3$ , in the vicinity of  $\Sigma$  we may write  $B_{\alpha\beta} \sim H_{\alpha\beta i}Y^i$ . This gives

$$B_{\alpha\beta}B^{\widetilde{\alpha}\widetilde{\beta}} \sim H_{\alpha\beta i}H^{\widetilde{\alpha}\widetilde{\beta}}{}_{j}Y^{i}Y^{j}, \qquad (3.16)$$

leading to a potential matrix  $V_{ij} \sim C \lambda^2 H_{\alpha\beta i} H^{\alpha\beta}{}_j$ . A mass is produced if the D7 brane wraps a cycle which overlaps the cycle which supports the nonzero flux.

Whether such masses break 4D supersymmetry or not depends on the details of the fluxes involved. Using a complex basis in the compact dimensions, this mass can preserve supersymmetry if the corresponding flux is purely of (2,1) or (1,2) type, and breaks supersymmetry [39] if it contains fluxes of (3,0) or (0,3) type.

Since our interest is in tracking how masses depend on warping for states localized in warped regions, it is instructive to specialize the above analysis to the special case where the D7 brane is localized within such a region,  $^{15}$  analogously to our discussion of D3 branes. Suppose then that the warp factor is  $e^{A(y)} = e^{A_m} e^{A(y)}$  throughout  $\Sigma$ , where  $e^{A_m} \ll 1$  and  $e^{A(y)}$  integrates over the cycle to give a result which is O(1) (notice that this implies that the flux relevant for generating masses for D7 moduli is also nonzero in the highly warped region, which is not the generic case). In this case, using the explicit formula for the warped throat metric we find fluctuation  $G_{ij} \sim \lambda e^{2A_m}$  and  $V_{ij} \sim \lambda^2 e^{4A_m}$ . Thus the mass for the fields  $Y^i$  is

$$m_Y \sim e^{A_m} \sqrt{\lambda} \sim \frac{e^{A_m}}{\mathcal{V}^{1/3}} \,,$$
 (3.17)

which coincides with the ones obtained in the previous section for the geometric moduli of a D3 brane.

#### 3.3 Effective 4D description

We now record what would be the corresponding description of such flux-induced masses for the brane geometric moduli from the point of view of the low-energy 4D theory. As emphasized earlier, this is possible if the SUSY breaking fluxes are chosen to be sufficiently small that it does not pay the lightest KK gravitino to become localized into the throat. Because we explicitly consider fluxes localized in a warped throat, the supersymmetry breaking sector lives in the warped region and the SUSY breaking scale is warped. We consider in turn the cases where the flux preserves and breaks supersymmetry, and these correspond to the two cases discussed above in sections 2.3.1 and 2.3.2. Our main interest is in how the overall warp suppression factor,  $e^{A_m}$ , appears in the low-energy theory.

<sup>&</sup>lt;sup>15</sup>For an example of such a construction, see [40].

# 3.3.1 Unbroken supersymmetry

If the relevant flux does not break N=1 4D supersymmetry then the effective 4D theory is described by the standard supergravity lagrangian, within which the D-brane moduli are represented by complex scalars residing within chiral supermultiplets. More precisely, the D7 moduli are described by a single complex scalar Y, whereas the six geometric D3 moduli are arranged into three complex scalars,  $\tilde{Y}^a$ , with a=1,2,3. We will discuss both kind of moduli under the same context.

The appropriate choice of Kähler potential is found by inspecting the kinetic terms for the D3 and D7 moduli, leading to the form  $K = 2A_m + \mathcal{K}$ , with

$$\mathcal{K} = K_c(\phi, \phi^*) + K_Y(\phi, \phi^*) Y^* Y + K_{\tilde{V}_a}(\phi, \phi^*) \tilde{Y}^{a*} \tilde{Y}^a + \cdots,$$
(3.18)

where  $\phi$  collectively denotes all the other moduli and the ellipses denote terms involving higher powers of  $\tilde{Y}^a$ , Y and their complex conjugates. We include an overall constant  $2A_m$ , as was argued above to be required in order to generically warp all nonzero masses for localized states by a factor  $e^{A_m}$ . This additive factor does not affect the  $\tilde{Y}^a$  and Y kinetic terms, however, so agreement with the microscopic calculation requires we also choose

$$K_Y = |\mu_7| \ k(\phi, \phi^*) \qquad K_{\tilde{V}^a} = |\mu_3| \ k^a(\phi, \phi^*),$$
 (3.19)

with no  $A_m$  dependence. For the present purposes the functions  $K_c$ , k and  $k^a$  could be arbitrary, although they are known explicitly for specific types of compactifications [41, 43, 44].

In this case the superpotential is *not* given only by the GVW form, since it also acquires a dependence on the D7 moduli [38, 44] (see also [45, 46]), which we expand to lowest order in Y:

$$W = W_{GVW}(\phi) + \frac{\mu_7}{2} w(\phi) Y^2 + \cdots$$
 (3.20)

As for D7 branes in the unwarped regions, we will assume that w does not carry any factors of the small quantity  $e^{A_m}$ .

The corresponding Kähler derivatives become

$$D_{\tilde{Y}^a}W = \partial_{\tilde{Y}^a}W + W\partial_{\tilde{Y}^a}K = \mu_3 k^a \tilde{Y}^{a*}W_{GVW} + \cdots, \qquad (3.21)$$

$$D_Y W = \partial_Y W + W \partial_Y K = \mu_7 \left[ w Y + k Y^* W_{GVW} \right] + \cdots, \qquad (3.22)$$

which show that  $\tilde{Y}^a = Y = 0$  does not break supersymmetry (or perturb the vacuum away from vanishing potential V).

In this case, keeping in mind the no-scale nature of the low-energy theory which ensures that  $W_{GVW} = V = 0$  at the minimum, the scalar mass term for the brane moduli is given by the contribution

$$V = e^{K} \sum_{\Phi = Y, \tilde{Y}^{a}} \frac{|D_{\Phi}W|^{2}}{K_{\Phi}} + \dots = |\mu_{7}| e^{2A_{m}} e^{K} \frac{|wY|^{2}}{k} + \dots,$$
(3.23)

Notice that the Y mass term now scales in the same way as it did in the microscopic computation, whereas on the other hand, the D3 moduli remain massless, in agreement with the no-scale structure of the potential.

#### 3.3.2 Broken supersymmetry

Next consider the case where the mass-generating flux breaks supersymmetry. Since the supersymmetry-breaking field is not within the low-energy theory, and since the mass splitting generated is much smaller than the generic KK mass, in this instance we expect the effective 4D theory to be described by a 4D supergravity supplemented by soft-breaking terms.

In this case the Y and  $\tilde{Y}^a$  kinetic terms are unchanged from the supersymmetric case, and the additive term for K is also required to ensure that all generic masses are warped by a factor of  $e^{A_m}$ , and so  $K = 2A_m + \mathcal{K}$ , with  $\mathcal{K}$  given by eq. (3.18). The additional suppression of the D7 modulus mass is then described by the relevant soft-breaking mass terms in the scalar potential. Specializing the result given in eq. (2.55) to the case of a diagonal metric, and canonically normalizing the kinetic terms, gives the following result for the physical Y mass [6]

$$m_Y^2 = m_{3/2}^2 - \mathcal{F}^m \mathcal{F}^{\overline{n}} \partial_m \partial_{\overline{n}} \ln K_Y , \qquad (3.24)$$

where we use  $V_0 = 0$ , and as before  $m_{3/2} = e^{K/2}|W|$  and  $\mathcal{F}^m$  is given by (2.54). Notice that both terms of (3.24) include a factor of  $e^{2A_m}$ , so that the soft-breaking mass of Y indeed has a factor of  $e^{A_m}$ .

Regarding the D3 moduli  $\tilde{Y}^a$ , due to the no-scale structure of the scalar potential for these vacua, the gravitino mass is generically related to  $K_{\tilde{Y}^a}$  in such a way that

$$m_{3/2}^2 = \mathcal{F}^m \mathcal{F}^{\overline{n}} \partial_m \partial_{\overline{n}} \ln K_{\tilde{Y}^a}$$
 (3.25)

and the soft masses for the D3 brane moduli vanish, even though supersymmetry is being broken.

In general the no-scale structure of the scalar potential is however spoiled by both  $\alpha'$  corrections and non-perturbative corrections to the superpotential, and the relation (3.25) will not hold. In that case, the soft masses (2.55) for the  $\tilde{Y}^a$  scalars become

$$m_{\tilde{Y}^a}^2 = V_0 + m_{3/2}^2 - \mathcal{F}^m \mathcal{F}^{\overline{n}} \partial_m \partial_{\overline{n}} \ln K_{\tilde{Y}^a} , \qquad (3.26)$$

where we have taken again a diagonal metric and canonically normalized kinetic terms. The value of the potential,  $V_0$ , now is also generically different from zero and given by the formula

$$V_0 = e^K \left( |DW|^2 - 3|W|^2 \right) . {(3.27)}$$

Thus, from the scaling  $e^K \propto e^{2A_m}$  we find that in scenarios with broken no-scale structure the soft masses for the D3 moduli are also suppressed by a  $e^{A_m}$  factor, in agreement with the results obtained in section 3.1 from a microscopic point of view.

#### 4. Towards phenomenology

Clearly our results may have interesting phenomenological implications. The study of soft supersymmetry breaking in the KKLT scenario [47, 48] has not included the effects of warping, while existing studies of warping effects in phenomenology break SUSY through brane

boundary conditions rather than fluxes (see e.g. [19]). Warping has only been considered in the mechanism for lifting to de Sitter space, but in this scenario we expect the standard model to appear from D-branes wrapping non-trivial cycles of the Calabi-Yau manifold, and a natural possibility is that these D-branes lie in a strongly-warped region (for explicit constructions in this direction see [49, 40]). This was actually part of the original motivation in [2], since the redshift in a throat is a natural mechanism to solve the hierarchy problem if the standard model sector lives on the tip of the throat. If the TeV scale supersymmetry breaking throat is regarded in the dual picture as a strongly-interacting CFT, then this can be regarded as a stringy realization of Witten's proposal for understanding the gauge hierarchy using dynamical supersymmetry breaking [50].

The substantial red-shifting due to the warp factor dependence of  $M_s^w$  implies that we can consider different scenarios depending on the corresponding warped string scale which can take any value between the electroweak scale (1 TeV) and the GUT scale ( $\sim 10^{17}\,\mathrm{GeV}$ ). On the other hand, all the other relevant scales are also suppressed by the same warp factor (and bulk volume dependence). As mentioned before, we expect the warped Kaluza-Klein scale  $M_{KK}^w$  to take values somewhat smaller than  $M_s^w$  due to the characteristic curvature scale  $1/\rho$  of the tip of the throat. Furthermore, the gravitino mass and all soft supersymmetry breaking terms are further reduced by factors controlling the amplitude of the supersymmetry-breaking flux,  $\vartheta, \vartheta'$ . Choosing these quantities very small, such as by localizing the sources of supersymmetry breaking into the warped throat allows the gravitino mass to be hierarchically smaller than  $M_{KK}^w$ . Generically the lightest gravitino is then localized in the throat, and describes a resonance in the dual CFT and is not naturally well-described by a low-energy 4D supergravity. However, if the couplings of the SUSY breaking physics in the throat to the gravitino are sufficiently weak, it may not pay the lowest mass gravitino KK mode to localize, and a low-energy supergravity action with standard soft supersymmetry breaking terms can be obtained. Notice that when a 4D supersymmetric description is possible, small fluxes imply a small value for the effective superpotential  $W_0$ . (Although a very small  $W_0$  is also required in the original KKLT scenario, this is for a very different reason: to have tree-level and non-perturbative contributions to the potential to compete, and to justify neglecting of the perturbative corrections to the Kähler potential.)

For  $M_{KK}^w \sim M_s^w$  at the GUT scale, we require  $f' \sim 10^{-13}$  to get a TeV gravitino. (An unwarped proposal which resembles this scenario is discussed in [47].) It is not clear that such scales can be understood in a controlled approximation, however, because the conditions  $e^{-4A_m} \gg \mathcal{V}^{2/3} \gg 1$  and  $M_s^w/M_p \sim e^{A_m}/\mathcal{V}^{1/3} \sim 10^2$  then require a relatively small volume. Smaller warped string scales arise more naturally in our scenario, including two potentially attractive possibilities. Having the warped string scale at the intermediate scale,  $M_s^w \sim 10^{11} \, \text{GeV}$ , could permit warped realizations of intermediate-scale string scenarios [51], which are also attractive from the point of view of some string inflationary models [4]. Getting a TeV gravitino mass in such models typically requires  $f' \sim 10^{-7}$ , which puts  $W_0$  in the range of validity of the KKLT approximations.

Alternatively, if the warped string scale were of order  $M_s^w \sim 10 \,\text{TeV}$  then  $f' \sim 1/10$ , would still justify the use of effective field theory. Since statistically speaking a very small

value of  $W_0$  is not preferred, one might argue that having such a low warped string scale is a more natural scenario to have. This would indicate a very interesting phenomenological scenario with a very small string scale and approximately supersymmetric effective action with soft SUSY breaking terms:

$$M_{1/2} \sim m_0 \sim A \sim m_{3/2} \sim \frac{1}{10} M_s^w \sim 1 \,\text{TeV}$$
 (4.1)

As mentioned earlier, this can be considered as a stringy realization of the dual of the dynamical supersymmetry breaking [50] approach to the hierarchy problem, with the geometrical picture of the exponential hierarchy being obtained from the warped geometry, rather than from strongly coupled dynamics in the dual conformal field theory.

For  $W_0 \sim 1$  we know that the KKLT approximations fail. In a large class of models [52] an exponentially large volume stabilization is obtained by including perturbative corrections to the Kähler potential. But for very large volumes the effects of warping are less and less important because the condition  $e^{-4A_m} \gg \mathcal{V}^{2/3}$  becomes more difficult to satisfy. Interestingly enough, in our set-up, values of  $W_0 \gtrsim 1$  would imply a collapse of the supersymmetric field theory approximation since the gravitino mass would be as heavy as the KK and string modes. In this case we do not expect an effective supersymmetric 4D action to play any role and we may have to consider directly an effective string theory phenomenology with distinctive signatures as compared with standard effective field theories, such as the presence of towers of KK and string states, etc. Alternatively, one might describe this situation in terms of the AdS/CFT dual.

One might foresee further scenarios developing depending on the location and the source of supersymmetry breaking as compared to the standard model.

# 5. Conclusions

In this paper we give a first step towards understanding the effective description of broken supersymmetric theories in strongly warped throats. Warped compactifications are very natural in IIB string theory [2, 53, 54] and provide a very rich, local and stringy scenario to discuss supersymmetry breaking with potentially different properties from standard scenarios of supersymmetry breaking in terms of gravity, gauge and anomaly mediation.

A typical compactification may have many throats and depending on the structure of the fluxes on each throat, the physics in each of the throats can be very different. Therefore within one single compactification we may have local models that feel differently the scale of supersymmetry breaking and therefore different structure of soft-breaking terms. In a large class of models this may not be describable in terms of effective 4D supergravities since the gravitino mass will be degenerate with the string and KK scales.

If the flux superpotential is small enough, a natural hierarchy is generated between the KK scale and the gravitino mass, justifying a supersymmetric effective field theory treatment. The exponentially large warp factor is a natural source of hierarchy as in the Randall-Sundrum model. In our case it provides the exponentially small scale of SUSY breaking instead of dynamical SUSY breaking. Our investigation allowed us to solve a puzzle regarding what defines 'the' gravitino of the low-energy 4D description. The natural notion from the extra-dimensional point of view is the lightest KK mode of the higher-dimensional gravitino, since this gauges the least broken 4D supersymmetry in the problem. This is also the state which is adiabatically linked to the massless gravitino in the supersymmetric limit. However, in strongly warped geometries this state prefers to be localized deep within the warped throat, and as a result generically couples to other states with interactions that are suppressed by the warped scale, rather than the 4D Planck scale. Such a state can be understood as a spin-3/2 resonance of the strongly interacting CFT which provides the dual description of the throat. Indeed it is generically not possible to capture the low-energy limit of such a system purely with a 4D (possibly softly broken) supergravity.

However there are situations where this is possible, most notably when supersymmetry breaking is localized within the throat but couples weakly enough that it does not make it energetically worthwhile for the lightest gravitino to localize there. In this case the 4D gravitino is most easily identified in the dual theory by perturbing away from the supersymmetric limit. Our analysis also leads us to suggest that the effective Kähler potential for such a system is shifted by the warp function  $A_m$  at the tip of the throat, but a general interpolation between the regimes of weak and strong warping is unknown.

Finally, we identify several potentially interesting phenomenological scenarios depending on the amount of warping and the tuning of the flux superpotential  $W_0$ . Further investigation of the detailed phenomenology of these new scenarios is certainly desirable. We also expect potential applications for cosmology, and in particular for inflationary scenarios depending on the existence of warping.

# Acknowledgments

We thank L. Ibáñez for collaboration in the early stages of this work. We acknowledge useful conversations on the subject of this paper with S. Abdussalam, J. Conlon, D. Cremades, O. DeWolfe, S. Hartnoll, S. Kachru, P. Kumar, S. Thomas, E. Silverstein, A. Sinha, and H. Verlinde. One of us (AM) would like to especially thank A. Frey for many useful discussions. CB's research is supported in part by funds from Natural Sciences and Engineering Research Council of Canada, the Killam Foundation and McMaster University. SBG and AM acknowledge the support of the Department of Energy under Contract DE-FG02-91ER40618. The work of PGC was supported by the EU under the contracts MEXT-CT-2003-509661, MRTN-CT-2004-005104 and MRTN-CT-2004-503369. KS wishes to thank Trinity College, Cambridge for financial support. C.B. thanks the kind hospitality of the Galileo Galilei Institute, where some of this work was done. SdA, SBG, FQ thank KITP Santa Barbara and the organizers of the workshop on 'String Phenomenology' for the same reasons, and acknowledge the partial support of the National Science Foundation under Grant No. PHY99-07949. FQ is partially funded by PPARC and a Royal Society Wolfson award. SdA wishes to thank the Perimeter Institute and DOE grant No. DE-FG02-91-ER-40672 for partial support.

# A. Warping and degenerate perturbation theory

In this appendix we use the example of a bulk scalar in the Randall-Sundrum scenario [14] as a toy model to discuss some features of KK reduction in highly warped regions. The KK reduction can be carried out explicitly in this model, thus the model is helpful for building intuition. We use it to examine the common practice of identifying supersymmetry-breaking mass shifts by truncating the KK reduction using supersymmetric configurations, and show when this can be justified in terms of a perturbative analysis. We show in particular why this analysis breaks down in the presence of strong warping.

#### Explicit diagonalization

Reference [14] considered a massive 5-d scalar

$$S = \int d^5x \sqrt{g} (-\partial_M Y \partial^M Y - M^2 Y^2)$$
 (A.1)

in a finite domain of AdS (of radius R)

$$ds^{2} = \frac{r^{2}}{R^{2}} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + \frac{R^{2}}{r^{2}} dr^{2}, \quad r_{0} < r < R$$
(A.2)

with Neumann boundary conditions at both ends.

Before discussing the KK reduction, we note that the above model has the essential features to capture the dynamics of the gravitino in IIB constructions in the highly warped regime ( $c \ll e^{-A_m}$ ). In the throat regions of IIB constructions, the metric typically factorizes to an  $AdS_5 \times X_5$  structure, for some manifold  $X_5$ , and so the wavefunction has a product structure. Thus, to understand the effects of warping it suffices to truncate to a 5D model. Also the ten dimensional mass term for the gravitino,  $e^{3A}G_{mnp}\gamma^{\widetilde{mnp}}$  is approximately a constant in the throat region<sup>16</sup> and typically scales as the inverse of the local AdS radius. Thus the simplest model to consider is that of a minimal massive scalar (A.1) with  $MR \sim 1$ .

We give a brief outline of the results of explicit Kaluza-Klein reduction (for details see [14]). For dimensional reduction, consider the 5D equations of motion for the ansatz

$$Y(x,r) = \sum_{n} u_n(r)\phi_n(x)$$
(A.3)

with  $\partial_{\mu}\partial^{\mu}\phi_n=m_n^2\phi_n$  ( $m_n$  are the four dimensional masses) . This yields an equation for  $u_n$ 

$$-\frac{1}{R^4 r} \partial_r (r^5 \partial_r u_n) + \frac{r^2}{R^2} M^2 u_n = m_n^2 u_n . \tag{A.4}$$

A general solution to the differential equation (A.4) can be written in terms of Bessel functions of order  $\nu = \sqrt{4 + (MR)^2}$ ,

$$u_n(r) = \frac{N_n}{r^2} \left[ J_\nu \left( \frac{m_n R^2}{r} \right) + b_n Y_\nu \left( \frac{m_n R^2}{r} \right) \right]$$
 (A.5)

<sup>&</sup>lt;sup>16</sup>For a discussion see section 2.2.2 and related discussion of the dilaton mass term in section 2.1.1.

where  $N_n$  and  $b_n$  are constants. The masses  $m_n$  and the constants  $b_n$  are fixed by the boundary conditions at  $r_0, R$ . For large warping  $(r_0/R \ll 1)$  one finds that the masses are determined by the equation

$$2J_{\nu}(x_n) + x_n J_{\nu}'(x_n) = 0 \tag{A.6}$$

where  $x_n = \frac{R^2}{r_o} m_n$ . In the regime  $RM \sim 1$ , the lowest root of (A.6) is of the order of unity and

$$m_0 \sim \frac{r_0}{R} M. \tag{A.7}$$

The wavefunction (A.5) is highly localized in the region close to  $r_0$ .

# Perturbative analysis

It is illustrative to examine this result in the context of perturbation theory, since it provides a toy example which shares many of the features which commonly arise when supersymmetry is broken in an extra-dimensional model. In supersymmetric models it is often useful to imagine turning on supersymmetry breaking in a parametrically small way, such as by turning on a small flux. In this case our interest is often in the size of the SUSY-breaking mass splittings which arise, as computed perturbatively in the SUSY-breaking parameter. This kind of problem has an analogue in the present example in the limit of small M, since the model acquires a new symmetry in this limit corresponding to shifts of the form  $Y \to Y + \epsilon$ . We therefore now consider computing the scalar mass in the small-M limit, in order to compare the results obtained perturbatively with the exact results found above. The perturbative methods that we discuss can also have applications to situations where an explicit diagonalization is not possible.

To proceed we treat the term involving the five dimensional mass M in (A.4) as a perturbation. We take the masses and wavefunctions of the unperturbed "hamiltonian" to be  $\mu_n$  and  $v_n$ , i.e

$$-\frac{1}{R^4 r} \partial_r (r^5 \partial_r v_n) \equiv H_0 v_n = \mu_n^2 v_n. \tag{A.8}$$

In the absence of the five dimensional mass term, the lowest mode is massless ( $\mu_0 = 0$ ) and has a constant wavefunction. For the higher modes, the masses and wavefunctions ( $\mu_i, v_i \ i > 0$ ) are given by (A.5) and (A.6) with  $\nu = 2$ . We note that the first roots of (A.6) are of the order of unity, hence the scale of the unperturbed KK tower is

$$m_{KK} \sim \frac{r_0}{R^2}. (A.9)$$

To set up the perturbative computation, we begin by introducing an inner product

$$\langle a|b\rangle = \int_{r_0}^{R} dr \ ra(r)b(r)$$
 (A.10)

under which the unperturbed Hamiltonian,  $H_0$ , for the KK states is hermitian. We normalize  $v_n$  as

$$\langle v_n | v_m \rangle = \delta_{nm}.$$
 (A.11)

Given the perturbation Hamiltonian

$$H' \equiv \frac{r^2}{R^2} M^2,\tag{A.12}$$

the mass of the lowest mode in first order non-degenerate perturbation theory is

$$m_0^2 = \langle v_0 | H' | v_0 \rangle. \tag{A.13}$$

With the normalization (A.11),

$$v_0 \sim \frac{1}{R}.\tag{A.14}$$

Then (A.13) gives

$$m_0 \sim M.$$
 (A.15)

This is the analogue of supergravity calculation which estimates the lowest gravitino KK mass by evaluating the supersymmetry-breaking action using the Killing spinor which defines the wavefunction of the mode which is massless in the supersymmetric limit.

Note that the perturbative result, eq. (A.15), is much larger than the correct value of the mass of the lowest mode (A.7). First order non-degenerate perturbation theory fails because the strength of the perturbation is large compared to the KK scale mass (A.9). Under such a circumstance one expects the wavefunction of the lowest excitation  $(u_0)$  to be significantly different from the lowest mode in the absence of the perturbation due to mixings between the zero mode  $(v_0)$  and the KK modes  $(v_i)$  introduced by the perturbation. These mixings are not captured by first order non-degenerate perturbation theory which does not incorporate the corrections to the wavefunction.

In situations where the KK scale is small compared to the strength of the perturbation, a better perturbative tool is degenerate perturbation theory, since effectively the KK modes  $(v_i)$  and the zero mode  $(v_0)$  are degenerate compared to the scale of the perturbation. Typically, one has to include a large number of KK modes to obtain quantitatively reliable results. Since our purpose here is to be illustrative we will include just one mode. We shall see that this is sufficient to reproduce correct estimates.

For the purposes of estimate, we approximate the first KK mode by its power law behavior; then with the normalization condition (A.11)

$$v_1 \sim \frac{r_0^3}{r^4}.$$
 (A.16)

In the subspace spanned by  $v_0$  and  $v_1$ , the perturbation H' has approximate matrix elements

$$\begin{pmatrix} M^2 & \frac{r_0^3}{R^3} \ln(R/r_0) M^2 \\ \frac{r_0^3}{R^3} \ln(R/r_0) M^2 & \frac{r_0^2}{R^2} M^2 \end{pmatrix}.$$
 (A.17)

The lowest eigenvalue of the perturbation matrix (A.17) is of the order of  $\frac{r_0^2}{R^2}M^2$ , which is in agreement with (A.7).

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